

Formula in Anselin, Luc. "Local Indicators of Spatial Association–LISA," *Geographical Analysis* 27(2): 93–115, 1995.

Local Moran's I:

$$I_i = (z_i/m_2) \sum_j w_{ij}z_j,$$
$$m_2 = \sum_i z_i^2/n$$

In our normal calculate : $z_i = x_i - \bar{X}$

Then:

$$I_i = \frac{x_i - \bar{X}}{m_2} \sum_j^n w_{ij} (x_j - \bar{X})$$
$$\boxed{m_2} = \frac{\sum_i^n (x_i - \bar{X})^2}{n}$$

Formula in Mitchell, Andy. *The Esri Guide to GIS Analysis, Volume 2*. Esri Press, 2005.

Local Moran's I, calculated for each feature (i)

The mean value (\bar{x}) is subtracted from the value of the neighbor (x_j) and the difference multiplied by the weight (w_{ij}) for the target-neighbor pair; the results for all neighbors are summed....

$$I_i = \frac{(x_i - \bar{x})}{s^2} \cdot \sum_j w_{ij} (x_j - \bar{x})$$

....then the sum is multiplied by: the difference between the mean value (\bar{x}) and the target feature value (x_i), divided by the variance (s^2)



$$s^2 = \frac{\sum_i^n (x_i - \bar{X})^2}{n} \quad \text{or} \quad = \frac{\sum_i^n x_i^2}{n} - \bar{X}^2$$

because of: $Var[X] = E(X^2) - (E(X))^2$

Formula in ArcGIS

The Local Moran's I statistic of spatial association is given as:

$$I_i = \frac{x_i - \bar{X}}{S_i^2} \sum_{j=1, j \neq i}^n w_{i,j} (x_j - \bar{X}) \quad (1)$$

where x_i is an attribute for feature i , \bar{X} is the mean of the corresponding attribute, $w_{i,j}$ is the spatial weight between feature i and j , and:

$$S_i^2 = \frac{\sum_{j=1, j \neq i}^n (x_j - \bar{X})^2}{n - 1} - \bar{X}^2 \quad (2)$$

with n equating to the total number of features.

These three parameters m_2 in Anselin 1995, S^2 in Mitchell 2005 and this S_i^2 in ArcGIS are supposed to be the same parameter. But this S_i^2 is not correct. Please see the next page!

Formula in Mitchell 2005 or Anselin 1995

$$s^2 = \frac{\sum_i^n (x_i - \bar{X})^2}{n}$$

or

$$= \frac{\sum_i^n x_i^2}{n} - \bar{X}^2$$

Formula in ArcGIS

$$S_i^2 = \frac{\sum_{j=1, j \neq i}^n (x_j - \bar{X})^2}{n - 1} - \bar{X}^2$$

$$S_i^2 = \frac{\sum_{j=1, j \neq i}^n (x_j - \bar{X})^2}{n - 1} - \bar{X}^2$$

The formula in ArcGIS is not the same as a normal formula of variance.

Either behind the sum have one more $-\bar{X}^2$, or in the parentheses have one more $-\bar{X}$!!

Is there any special mathematical sinn or is that just wrong??

Is the calculation in the toolbox based on this wrong formula also injustice?